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### International Journal of Polymeric Materials

Publication details, including instructions for authors and subscription information: <http://www.informaworld.com/smpp/title~content=t713647664>

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To cite this Article Dutta, A. and Ryan, M. E.(1982) 'Parison Inflation in Extrusion Blow Molding: A Theoretical Analysis for Identifying Critical Process Parameters', International Journal of Polymeric Materials, 9: 3, 201 — 215 To link to this Article: DOI: 10.1080/00914038208077980 URL: <http://dx.doi.org/10.1080/00914038208077980>

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*Intern. J. Polymeric Mater.,* **1982, Vol. 9,** pp. **201-215 00914037/82/09034201** *%06.50/0 0* **1982** Gordon and Breach Science Publishers, Inc. Printed in Great Britain

# Parison Inflation in Extrusion Blow Molding: **A** Theoretical Analysis for **Identifying Critical Process** Parameters

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*(Received August 27,1981)* 

An analysis for describing parison (cylindrical) inflation behavior in the extrusion blow molding process is presented. A general growth equation is developed starting from the basic conservation principles. Assuming the polymer melt constituting the parison to behave as a purely viscous Generalized Newtonian Fluid, the effect of different process and material parameters on the inflation process is investigated. From the numerical results, it is inferred that the growth behavior for inelastic liquid exhibits a general tendency of approaching exponential (constant stretch rate) growth as elapsed time progresses. Besides, the initial parison dimensions are determined to play a very significant role in governing the inflation process. Moreover, the inertial contribution owing to fluid motion is found to exert an appreciable influence on the growth dynamics, and hence cannot be neglected without introducing severe approximations in the analytical development.

#### **INTRODUCTION**

Extrusion blow molding is one of the most commonly used polymer forming operations for production of hollow containers. The continued popularity and importance of this process **is** evident from the enormous growth and diversification of the plastics container industry in recent years. In spite of its considerable growth, however, there is a remarkable paucity of fundamental understanding pertaining to the operation of the entire blow molding cycle. Generally, the blow molded product is subjected to severe constraints with regard to its weight and wall thickness distribution. One of the major factors

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governing the thickness variation (of the product) is the thickness profile of the extruded parison. Consequently, a significant portion of the research efforts associated with the analysis of the extrusion blow molding process has focused primarily on the various aspects of parison formation, such as optimum die design, $1-3$  relationships between the material properties, operating conditions, and parison quality, $4^{-10}$  and techniques for characterizing parison swell and draw down behavior.<sup>11-15</sup> In addition to the parison quality, however, the behavior of the parison during subsequent stages of inflation and cooling is also of significant importance for efficient operation of the entire process. Therefore, it follows, that a detailed knowledge regarding all the various stages involved in a blow molding cycle is essential to provide a reliable scientific basis for optimizing process conditions and controlling product quality.

The present investigation deals with the inflation dynamics of a cylindrical parison. Currently, very little information is available regarding the particular nature in which the parison inflates. Denson<sup>16</sup> has briefly discussed the problem of parison inflation in his review of extensional flows in polymer processing. He employed a simple analysis for a power-law liquid in order to illustrate the application of "phenomenological equations of state" for the analysis of practical situations involving extensional flows. **A** similar approach has also been suggested by Middleman<sup>17</sup> in his treatise on polymer processing. In both cases, the inertial contributions were neglected. However, parison inflation is a very rapid process, and it is quite likely that the inertia of the fluid will have a significant influence on the growth dynamics. This aspect, indeed, is more clearly demonstrated in the course of the subsequent discussion.

In the following sections, a theoretical analysis is developed in order to provide some insight into the complex problem of parison inflation by considering the analogous situation of the radial growth of a thin cylindrical shell due to constant internal pressure. Clearly, this idealized situation implies that the end effects are considered negligible. This, however, is deemed to be an appropriate assumption considering the large length to diameter ratio of the parisons commonly encountered in industrial practice.

#### **ANALYSIS**

We consider a cylindrical parison of constant length, shown schematically in Figure 1, being subjected to a constant applied pressure difference,  $\Delta p = p_i - p_a$ , where  $p_i$  and  $p_a$  are the pressures inside and outside the parison respectively. It is assumed that the melt constituting the parison is incompressible and that the process occurs under isothermal conditions. In addition, the growth of the parison due to the imposed pressure difference is taken to



**FIGURE 1 Coordinate system used for inflation analysis.** 

occur axisymmetrically. Consequently, our primary objective is to determine the radius, *R,* as it increases with time, *t,* and also the dependence of this growth behavior on different material properties and process conditions.

#### **1. General growth equation**

Since the parison is assumed to inflate only radially, the only velocity component present is the radial velocity,  $v_r = v_r(r, t)$ . Thus, the equation of continuity and the r-component of the equation of motion can be simply written as

$$
\frac{\partial}{\partial r}(rv_r) = 0 \tag{1}
$$

and

$$
\rho \left( \frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} \right) = \frac{\partial}{\partial r} \left( -p + \tau_{rr} \right) + \frac{\tau_{rr} - \tau_{\theta \theta}}{r}
$$
(2)

where  $\rho$  is the density, p the pressure,  $\tau_{rr}$  and  $\tau_{\theta\theta}$  are the extra stresses in radial and circumferential directions respectively. Since  $v_r(R, t) = dR/dt = \dot{R}$ , integration of Eq. (1) gives the radial velocity as

$$
v_r = R\dot{R}/r \tag{3}
$$

The rate of deformation tensor,  $\Delta$ , can therefore be expressed as

$$
\Delta = \hat{\epsilon} \begin{pmatrix} -2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{pmatrix}
$$
 (4)

where  $\dot{\varepsilon} = R\dot{R}/r^2$  is the extension rate. The second scalar invariant of  $\Delta$  is then given by

$$
II_{\Delta} = \Delta : \Delta = 8\dot{\varepsilon}^2 \tag{5}
$$

The form of the rate of deformation tensor implies that the flow is planar extensional in nature. This existence of planar extensional behavior for parison (cylindrical) inflation in a blow molding process has been indicated by Denson.<sup>16</sup>

The initial and boundary conditions for this flow situation can be represented as follows :

$$
-(-p+\tau_{rr})|_{r=R}+\sigma/R=p_i \quad \text{at} \quad r=R, \quad t>0 \tag{6a}
$$

$$
-(-p+\tau_{rr})|_{r=S}-\sigma/S=p_a \quad \text{at} \quad r=S, \quad t>0 \tag{6b}
$$

$$
\dot{R} = \dot{S} = 0 \quad \text{for} \quad t \leq 0 \tag{6c}
$$

$$
R = R_0, \quad S = S_0 \quad \text{for} \quad t \leq 0 \tag{6d}
$$

where  $\sigma$  is the surface tension of the liquid, S is the external radius of the parison, and the subscript 0 indicates initial values, that is, at  $t = 0$ . The boundary conditions given by Eqs. (6a) and (6b) are obtained from force balances at the internal and external surfaces of the parison assuming that the stresses developed in the gas phase can be considered to be negligible owing to their relatively small magnitude.

Substituting for the radial velocity,  $v_r$ , into Eq. (2) and integrating the resulting expression over the entire parison thickness  $(r = R \text{ to } r = S)$  yields

$$
\rho \left[ (R\ddot{R} + \dot{R}^2) \ln S/R - \frac{\dot{R}^2}{2} \left( 1 - \frac{R^2}{S^2} \right) \right]
$$
  
=  $p_i - p_a - \frac{\sigma}{R} (1 + R/S) + \int_R^S \frac{\tau_{rr} - \tau_{\theta\theta}}{r} dr$  (7)

where  $\ddot{R} = d^2R/dt^2$ . Normally the parison thickness,  $\delta$ , is much smaller than the radius of the parison, that is  $\delta/R \ll 1$ . Therefore, the following expressions

involving the ratio  $S/R$  can be approximated as

$$
(S/R)^m = (1 + \delta/R)^m \approx 1 + (m\delta/R)
$$
 (8a)

and

$$
\ln (S/R)^m \approx m\delta/R \tag{8b}
$$

where *m* is an integer. Making use of these approximations, Eq. (7) can be expressed as

$$
\Delta p - \frac{\sigma}{R} (2 - \delta/R) - \rho \dot{R} \delta = - \int_{R}^{S} \frac{\tau_{rr} - \tau_{\theta\theta}}{r} dr \tag{9}
$$

Eq. (9) is a general expression describing inflation of a cylindrical parison under a constant applied pressure difference  $\Delta_p$ , and is applicable for both purely viscous and viscoelastic liquids. Presently, however, we will restrict ourselves only to the inelastic case.

#### **II. Inelastic liquids**

Starting with Stokes' hypothesis and making use of the Cayley-Hamilton theorem, the relationship between the extra stress tensor,  $\tau$ , and the rate of deformation tensor,  $\Delta$ , for an inelastic liquid may be expressed as

$$
\tau = \eta \Delta + g \Delta^2 \tag{10}
$$

The material functions  $\eta$  and g are functions of the three scalar invariants ( $I_A$ ,  $II_A$ ,  $III_A$ ) of the rate of deformation tensor,  $\Delta$ . However, incompressibility implies that I<sub>A</sub> = 0 and Eq. (4) gives  $III_A = 0$ . The scalar functions,  $\eta$  and g, therefore depend only on the second scalar invariant,  $II_{\Delta}$ . Also,  $I_{\Delta} = 0$  implies that

$$
\tau_{rr} - \tau_{\theta\theta} = 2\tau_{rr} \tag{11}
$$

and Eq. (9) reduces to

$$
\phi = -2 \int_{R}^{S} \frac{\tau_{rr}}{r} dr = 4R\dot{R} \int_{R}^{S} \frac{\eta dr}{r^3} - 8R^2 \dot{R}^2 \int_{R}^{S} \frac{g dr}{r^5}
$$
(12)

where

$$
\phi = \Delta p - \frac{\sigma}{R} (2 - \delta/R) - \dot{R} \rho \delta \tag{13}
$$

If the stress tensor is assumed to be a linear function of the rate of deformation tensor, g = 0 (Generalized Newtonian Fluid) and Eq. **(12)** reduces to

$$
\phi = 4R\dot{R} \int_{R}^{S} \frac{\eta dr}{r^3} \tag{14}
$$

Alternatively, Eq. (14) can be written as

$$
\phi = -\frac{2\dot{R}}{R} \left[ \left( \frac{R}{S} \right)^2 \eta_s - \eta_R \right] + 2R\dot{R} \int_R^S \frac{\partial \eta}{\partial r} \frac{dr}{r^2}
$$
(15)

where  $\eta_s$  and  $\eta_R$  denote the function  $\eta(\text{II}_\Delta)$  evaluated at  $r = R$  and  $r = S$ respectively. Equation *(15)* can be used to determine the growth behavior of an inelastic liquid for which the scalar function,  $\eta(I_{\lambda})$  is known.

Two particular purely viscous rheological equations of state are now considered.

Power-law (Ostwald-de Waele) model :  $\eta = K\left|\frac{1}{2}(\Delta:\Delta)\right|^{(n-1)/2}$  (16)

Bingham plastic model :

$$
\eta = (\mu + \tau_y) \frac{1}{2} (\Delta : \Delta)|^{-1/2}) \quad \text{for} \quad \frac{1}{2} (\tau : \tau) > \tau_y^2
$$
  

$$
\Delta = 0 \quad \text{for} \quad \frac{1}{2} (\tau : \tau) < \tau_y^2 \tag{17}
$$

where K is the power-law constant, *n* the power-law index,  $\mu$  the viscosity, and  $\tau_{v}$  is the yield stress. Combining the viscosity function,  $\eta$ , with Eq. (15) and using the thin shell approximation given by Eq. *(8),* the growth equation for the power-law and the Bingham plastic models are expressed as follows :

Power-law: 
$$
\phi = 4K(2)^{n-1} \left(\frac{\dot{R}}{R}\right)^n \frac{C}{R^2}
$$
 (18)

Bingham Plastic: 
$$
\phi = \frac{2C}{R^2} \left[ 2\mu \frac{\dot{R}}{R} + \tau_y \right]
$$
 for  $\Delta_{\mathbf{P}} > \frac{\tau_y \delta_0}{R_0}$  (19)

where  $\delta_0 = S_0 - R_0$  and  $C = R_0 \delta_0 = R \delta$  is a constant dictated by material conservation.

It is convenient to reduce the number of independent parameters by nondimensionalizing the variables of interest. For that purpose, we define a characteristic process time  $\gamma = R_0(\rho/\Delta_p)^{1/2}$ . The time and spatial coordinate are then nondimensionalized as  $\theta = t/\gamma$  and  $\psi = R/R_0$  respectively. The dimensionless form of Eqs. (18) and (19) become

$$
\dot{\psi} = \frac{\psi}{C^*} - \frac{2}{C^* \text{ We}} + \frac{1}{\text{We } \psi^2} - \frac{2}{\beta \psi} \left(\frac{2}{\gamma} \frac{\dot{\psi}}{\psi}\right)^n \tag{20}
$$

and

$$
\dot{\psi} = \frac{\psi}{C^*} - \frac{2}{C^* \text{ We}} + \frac{1}{\text{We } \psi^2} - \frac{4}{\text{Re } \psi^2} - \frac{2\tau^*}{\psi} \quad \text{for} \quad \tau^* C^* < 1 \tag{21}
$$

where

$$
\text{Re} = \frac{\gamma \Delta_p}{\mu}, \qquad \text{We} = \frac{R_0 \Delta_p}{\sigma}
$$

$$
\beta = \Delta_{\mathbf{P}}/K, \qquad \tau^* = \tau_y/\Delta_{\mathbf{P}}, \qquad C^* = \delta_0/R_0
$$

and  $\dot{\psi}$ ,  $\dot{\psi}$  are the first and second derivatives of  $\psi$  with respect to  $\theta$ . In order to obtain the growth profile for a particular liquid model, the appropriate growth equation is solved for  $\psi(\theta)$  subject to the initial conditions  $\psi(0) = 1$ and  $\dot{\psi}(0) = 0$ .

The nonlinear form of Eqs. (20) and (2 1) precludes any reasonable possibility of an analytical solution except for some limiting cases. These growth equations, representing an initial value problem were therefore solved numerically by employing the fourth-order Milne predictor-corrector technique with an improvement of the predictor value using the local truncation error for faster convergence.<sup>18</sup> Details of the numerical procedure and the computer program has been provided elsewhere.<sup>19</sup>

#### **RESULTS AND DISCUSSION**

Growth profiles for both the power-law and Bingham plastic models were obtained for different conditions in order to investigate the influence of the various process and material parameters governing the inflation behavior. For most polymer melts the Weber number, We, is typically large in magnitude  $({\sim}10^{4}-10^{6})$ . Consequently, from the growth equations it can be expected that the contribution owing to the surface tension of the liquid will be much smaller as compared to these due to inertial and viscous effects. This behavior was indeed corroborated by the numerical results even for We having an assigned value as low as 10<sup>3</sup>. Thus, for all practical purposes, the effect of surface tension on the growth process can be assumed to be negligible and will be taken to be so during the course of the following discussion.

Figures 2 and 3 illustrate the growth behavior of a power-law liquid for varying degrees of pseudoplasticity (power-law index, n). Increase in pseudoplasticity gives rise to a faster growth of the parison. **A** more interesting aspect, probably, is that after a sufficient period of time has elapsed, the process assumes an almost exponential growth. The time required to achieve this limiting behaviour reduces as  $n$  decreases. Under these conditions the magnitude of the growth rate to the radius ratio,  $\dot{\psi}/\psi$ , may be expressed as follows

$$
\frac{\dot{\psi}}{\psi} = \left[\frac{1}{C^*} - \frac{2}{\beta} \left(\frac{2\dot{\psi}}{\gamma\psi}\right)^n \exp(-2\dot{\psi}\theta/\psi)\right]^{1/2} \tag{22}
$$

Clearly as  $\theta$  increases,  $\psi/\psi$  tends to the limiting value of  $C^{*-1/2}$  as evident in Figure 3. Also, note that  $\psi/\psi = \gamma \dot{\epsilon}_R$ , where  $\dot{\epsilon}_R$  is the extension rate at  $r = R$ . Therefore, it seems that as the parison inflates, the resulting planar extensional



**FIGURE 3 Time dependence of the growth rate to radius ratio for different power-law indices.** 

flow situation tends to achieve a motion with a constant stretch rate, at least at the internal surface of the parison. Theoretically, of course, this limiting condition will be attained only at infinite  $\theta$ . However, if we hypothesize that the time required to reach exponential growth,  $\theta_s$ , occurs at the time when  $\dot{\psi}/\psi$  =  $XC^{*-1/2}$ , then  $\theta_s$  can be estimated as

$$
\theta_s = \frac{C^*}{2X} \ln \left[ \frac{2C^{*(1-n)/2}}{\beta(1-X^2)} \left( \frac{2X}{\gamma} \right)^n \right] \tag{23}
$$

where  $X$  is an arbitrary value chosen to be very close to (but less than) unity. The influence of material properties and process conditions on  $\theta_s$  are summarized in Table I.

 $\beta$  and  $\gamma$  are two important parameters for parison inflation and their effects on the growth profiles are presented in Tables I1 and 111 respectively. Results in Table II indicate that faster inflation is achieved by increasing  $\beta$ ; but this enhancement in parison growth diminishes very rapidly as the magnitude of  $\beta$ itself increases. Besides, the conditions specified in Table II imply that  $\Delta_{\bf p}$ remains unchanged. Consequently, any effect owing to an increase in  $\beta$  can directly be attributed to a decrease in the power-law constant, *K.* On the other hand, results in Table III suggest that larger characteristic process time,  $\gamma$ , facilitates parison inflation. Here again, the conditions imply that changes in  $\gamma$ can only be possible if the liquid density,  $\rho$ , or the initial radius,  $R_0$ , changes. Therefore, assuming the density to be constant, it can be inferred that the curvature of the cylindrical parison is an inhibiting factor towards inflation. Alternatively, the larger the initial radius,  $R_0$ , the more readily will the parison inflate.

The importance of the geometric factor, *C\*,* in governing the growth dynamics was previously indicated. This interesting feature is now more clearly demonstrated in Figure 4. It is seen that an increase in  $C^*(\delta_0)$  to be more

			$\theta_{s}$		
		$\gamma = 0.1 \text{ sec}, \quad X^2 = 0.9999$	$C^* = 1/16$ , $X = 0.99$ $\beta = 5 \text{ sec}^{-1}$		
	$\beta = 3 \text{ sec}^{-n}$	$\beta = 5 \text{ sec}^{-1}$			
n	$C^* = 1/16$	$C^* = 1/16$ $C^* = 1/9$			$\gamma = 0.1$ sec $\gamma = 0.2$ sec
0.0	0.754	0.690	1.016	0.115	0.115
0.2	0.864	0.800	1.153	0.225	0.208
0.4	0.973	0.909	1.289	0.335	0.300
0.6	1.083	1.019	1.426	0.445	0.393
0.8	1.192	1.128	1.562	0.555	0.485
1.0	1.302	1.238	1.699	0.665	0.578

**TABLE I** 

**Time required to achieve exponential growth for power-law liquids** 

#### TABLE **I1**

We = $10^5$ , $\gamma = 0.1$ sec, $C^* = 1/16$ , $n = 0.6$									
β		0.5		2.0		5.0		10.0	
$sec^{-n}$ $\theta$	ψ	$\dot{\psi}/\dot{\psi}$	ψ	$\dot{\psi}/\dot{\psi}$	ψ	$\dot{\psi}/\dot{\psi}$	ψ	$\dot{\psi}/\psi$	
0.0	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000	
0.1	1.032	0.464	1.065	1.147	1.074	1.361	1.078	1.439	
0.2	1.088	0.578	1.243	1.907	1.296	2.343	1.316	2.498	
0.3	1.159	0.684	1.551	2.503	1.698	3.009	1.753	3.174	
0.4	1.249	0.824	2.042	2.987	2.348	3.434	2.460	3.565	
0.5	1.368	1.001	2.808	3.358	3.356	3.689	3.555	3.778	
0.6	1.530	1.244	3.983	3.618	4.892	3.835	5.219	3.889	
0.7	1.758	1.547	5.771	3.784	7.210	3.915	7.724	3.945	
0.8	2.090	1.930	8.470	3.883	10.69	3.957	11.48	3.973	
0.9	2.589	2.355	12.53	3.938	15.90	3.978	17.09	3.987	
1.0	3.349	2.796	18.60	3.968	23.68	3.989	25.47	3.994	

Effect of  $\beta$  on parison growth behavior

precise) results in a retarded growth. **As** a result, even though the growth rate to the radius ratio asymptotically approaches a magnitude of  $C^{*-1/2}$  for all values of  $C^*$ , this asymptotic limit is attained more rapidly as the initial parison thickness,  $\delta_0$ , is decreased. Hence, it appears that the initial parison dimensions,  $\delta_0$  and  $R_0$ , are two of the most critical process parameters controlling inflation behavior and play a central role in determining the requisite blowing time for the process.

**A** Bingham plastic liquid is considered next in order to ascertain the

TABLE **I11** 

We = $10^5$ , $\beta = 3 \text{ sec}^{-n}$ , $C^* = 1/16$ , $n = 0.8$								
γ sec θ	0.05		0.075		0.10		0.20	
	ψ	$\dot{\psi}/\psi$	ψ	$\dot{\psi}/\psi$	ψ	$\dot{\psi}/\dot{\psi}$	ψ	$\dot{\psi}/\dot{\psi}$
0.0	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000
0.1	1.053	0.873	1.060	1.014	1.063	1.101	1.070	1.262
0.2	1.173	1.256	1.207	1.550	1.228	1.737	1.269	2.090
0.3	1.352	1.586	1.441	1.991	1.499	2.241	1.614	2.690
0.4	1.613	1.947	1.797	2.424	1.919	2.696	2.162	3.142
0.5	1.999	2.351	2.339	2.845	2.566	3.098	3.013	3.470
0.6	2.582	2.764	3.168	3.214	3.557	3.419	4.313	3.690
0.7	3.470	3.144	4.436	3.504	5.069	3.650	6.284	3.827
0.8	4.829	3.499	6.366	3.706	7.361	3.800	9.255	3.907
0.9	6.898	3.669	9.286	3.834	10.82	3.891	13.71	3.951
1.0	10.03	3.811	13.68	3.910	16.01	3.942	20.38	3.975

Dependence of parison inflation behavior on the characteristic process time, *<sup>y</sup>*



**FIGURE 4 Effect of the geometric parameter, C\*, on parison inflation.** 

generality of the inflation characteristics exhibited by the purely viscous liquid. The inflation behavior of a Bingham plastic liquid for different yield stresses,  $\tau^*$ , is shown in Figures 5 and 6. Clearly, an increase in  $\tau^*$  retards parison growth. Also, the growth dynamics of the Bingham plastic is very similar to that exhibited by the power-law liquid, with  $\dot{\psi}/\psi$  asymptotically approaching  $C^{*-1/2}$  as time progresses. Furthermore, a special case of both the power-law  $(n = 1)$  and the Bingham plastic  $(\tau_v = 0)$  is the Newtonian liquid. Thus, for Newtonian liquid the growth Eqs. **(20)** and (21) reduce to

$$
\dot{\psi} = \frac{\psi}{C^*} - \frac{2}{C^* \text{ We}} + \frac{1}{\text{We } \psi^2} - \frac{4\dot{\psi}}{\text{Re } \psi^2}
$$
(24)

For the limiting case where surface tension effects may be neglected and the Reynolds number becomes very large (Re  $\rightarrow \infty$ ), Eq. (24) gives

$$
\psi = \cosh(\theta C^{*-1/2}) \tag{25}
$$

and

$$
\dot{\psi}/\psi = C^{*-1/2} \tanh(\theta C^{*-1/2}) \tag{26}
$$

Eq. (26) predicts that as  $\theta \to \infty$ ,  $\dot{\psi}/\psi \to C^{*-1/2}$ , similar to the predicted results



**FIGURE 6 Effect** of **yield stress** on **the growth rate to radius ratio for a Bingham plastic.** 

for the non-Newtonian liquids considered previously. **As** illustrated in Figure **7,** for a Newtonian liquid, the parison inflates more rapidly with an increase in Reynolds number, Re. In addition, it can be seen that for  $Re > 10$ , the growth behavior is not significantly different for that predicted by Eq. (25) for  $Re = \infty$ . Thus, as might be expected, an increase in the inflation pressure or a decrease in the viscosity of the melt will result in faster growth of the parison.

The significance of the inertial contribution to the inflation process is now discussed. The importance of fluid inertia can be adequately demonstrated by considering the growth of a power-law liquid as described by Eq. (20). **If,** as a simplification, we drop the inertial terms in Eq. (20), it reduces to the following simple form

$$
\dot{\psi}/\psi = \alpha \psi^{2/n} \tag{27}
$$

where  $\alpha = \gamma/2(\beta/2C^*)^{1/n}$  and, of course, surface tension of the liquid is neglected. Integration of **Eq.** (27) gives the resulting expression for the parison radius

$$
\psi = \left(\frac{n}{n - 2\alpha\theta}\right)^{n/2} \tag{28}
$$

Unlike the growth behavior discussed previously, **Eq. (28)** predicts that as



**FlGURE 7 Influence of Reynolds number on the growth. behavior of a Newtonian** liquid.



**FIGURE 8** Comparison of inflation behavior of a Newtonian liquid with  $($ -----) and without  $($ -----) inertial contribution.

 $\theta \rightarrow n/2\alpha$ ,  $\psi \rightarrow \infty$ . Hence, instead of approaching an asymptotic limit, the growth rate to radius ratio now tends to become infinite at a critical time determined by the process conditions and material properties. Also, Eq. **(27)**  suggests the occurrence of a non-zero initial rate of magnitude  $\alpha$ . This implies a sudden jump in the growth rate at the onset of inflation process and does not appear to be realistic. Figure **8** provides a comparison between the growth profiles with and without the inclusion of the inertial terms for the case of a Newtonian fluid ( $n = 1$ ,  $\alpha = \text{Re}/4C^*$ ). Significant differences in the growth characteristics are clearly evident. In the absence of inertial effects, inflation proceeds at a much greater rate and the entire process becomes unbounded within a short time interval.

#### **CONCLUSION**

In the preceding sections, an analysis of parison inflation (in extrusion blow molding) for inelastic polymer melts has been presented. The primary objective of the theoretical development is to illuminate some of the effects produced by changes in material properties and process conditions on the growth dynamics and to identify the critical parameters controlling the inflation behavior. For the inelastic liquids (Newtonian and non-Newtonian) considered in the present study, it is determined that they exhibit a general growth behavior of approaching exponential growth as elapsed time progresses. This limiting behavior can alternatively be interpreted as the constant rate planar extension of the internal surface of the parison. Interestingly, the magnitude of this exponential growth rate,  $\dot{\psi}/\dot{\psi}$ , is found to be dependent only on the geometric factor *C\** which, in turn, is related to the initial parison dimensions. Therefore, in conclusion, the initial parison dimensions appear to play a very significant role in governing the dynamics of parison inflation.

Additionally, apart from the negligible influence of the surface tension of the melt in determining the inflation process, the analysis suggests that faster inflation, commonly desired for industrial blow molding operation, can be achieved by (i) decreasing the melt viscosity (ii) decreasing the yield stress if the melt is Bingham plastic, and (iii) by increasing the inflation pressure. Finally, for rapidly occurring processes, such as parison inflation, the inertial contribution due to fluid motion is most likely to be appreciable and cannot be neglected without introducing severe approximations into the subsequent analysis.

#### **Acknowledgement**

The authors would like to express their appreciation to the State University of New York at Buffalo for financial support.

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